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Multivariable H_∞ Control of Wastewater Biological Treatment Processes

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Abstract: This paper has been carried out to develop an efficient multivariable H_∞ robust control system, in the presence of the bounded parametric uncertainties, with good disturbance and measurement noise compensation. This strategy was applied to a biological wastewater treatment process in order to control the organic substrate concentrations associated with an appropriate control of the dissolved oxygen concentration. The objective of this paper is to synthesize a H_∞ robust control structure with good results in reference tracking (in regard with the organic substrate and the dissolved oxygen concentration) and compensation of measurement noise, disturbances and model parametric uncertainties. The main disturbances were identified by using statistical precipitation data collected in multiple meteorological stations across the country. The literature considers only three main pluviometrical regimes: rain, normal and dryness. The wastewater treatment process with activated sludge is controlled considering the pluvial influent for an interval of one year and its daily variation.

Keywords: robust controller, wastewater treatment process, activated sludge, multivariable control.

1. INTRODUCTION

The biological wastewater treatment is extremely important from the environmental conservation and protection point of view, and this importance is also sustained by the protective active legislation. The main objective of the wastewater treatment process is to eliminate a series of pollutants such as the organic substrate concentration, nitrogen, phosphorous etc., so that the resulting water to fulfill the standard required by law and therefore to be able to be discharged in the natural receivers.

The biological wastewater treatment processes are complex nonlinear processes that are affected by disturbances and uncertainties. Basically, due to these concerns, the modeling and the control of these processes are difficult and they represent a real challenge for the teams of specialists interested in optimizing the wastewater treatment processes. The literature mentions numerous approaches for the modeling and the control of the biological wastewater treatment process, all of them having/serving the purpose of improving the operating regime and, of course, the effluent quality in steady-state and dynamic regimes.

(Jeppsson 1996) presents an overview of the wastewater treatment process with activated sludge modeling evolution. The first two dynamic models were proposed by (Goodman and Englande 1974; Buhr *et al.* 1974). The state variables, substrate and the biomass, were considered sufficient in the dynamic description of the process and the substrate degradation was modeled as a first order reaction (Eckenfelder and O'Connor 1955; McKinney 1962; Eckenfelder 1966). Later on, a Monod dependency of the removal rate on substrate concentration was introduced (Lawrence and McCarty 1970). Busby and Andrews (1975) have introduced one of the first structured models, in which the biomass was classified as active, settled and inert. The model proposed by (Dold *et al.* 1980) represents the fundament for further approaches in the mechanistic modeling of the wastewater treatment process with active sludge. In 1983, the International Water Association (IWA) initiated the implementation of practical models for designing and operating the wastewater treatment plants with activated sludge and in 1987 published a first condensed mathematical representation, ASM1 (Activated Sludge Model No. 1, Henze *et al.* 1987). The model uses 13 state variables which describe the biological removal of organic carbon and nitrogen. (Copp *et al.* 2002) demonstrated that similar results can be obtained by testing the ASM1 model on different software platforms. It is the first model accepted as reference (Dircks *et al.* 2001; Roeleveld and van Loosdrecht 2002) both by theoreticians and practitioners (Gernaey *et al.* 2004). The model has been extended further, considering the experimental observations (Sollfrank and Gujer 1991) and the information regarding the population growth and dynamics (Gujer and Kappeler 1992).

This model, that also describes the phosphorous elimination, was named ASM2 (Henze *et al.* 1995). Although it contains 19 state variables and 65 parameters, it still does not include all the phenomena observed. Further on, the IWA group proposed two new models, the ASM2d and ASM3 (Henze *et al.* 2000). The model reliability depends on an increased number of kinetic and stoichiometric parameters that are generally dependant on the wastewater influent characteristics, thus needing experimental calibration (Jeppsson 1996).

A simplified version of the ASM1 model has been proposed by (Nejjari 1999). This model has to deal only with four state variables: biomass, organic substrate, dissolved oxygen and recycled sludge. Some other modeling approaches of the wastewater treatment processes with activated sludge can also be mentioned: the dynamic "black box" type models and artificial intelligence-based techniques (fuzzy, neural, expert systems).

From the point of view of the available literature, the following control techniques can be mentioned: the conventional control techniques based on linear controllers (Marsili-Libelli 1992; Ayesa *et al.* 2006; Yoo and Kim 2009), the optimal control techniques (Buitron *et al.* 2004; Grigorieva and Khailov 2010), the predictive control techniques (Brdys and Zhang 2001), as well as the adaptive linearizing control techniques (Renard *et al.* 1988; Dochain and Perrier 1992). All these methods provide good performances if the process functions within certain points. Unfortunately, the wastewater treatment process is affected by changes within the operating point due to the pluviometric conditions, high variations of the influent substrate and so on, leading to the necessity of using robust control methods (e.g. H_∞ method, which is used in this paper), that provide superior results in comparison to those mentioned above.

The robust control deals explicitly with the process uncertainties in the controller synthesis, having an adequate behavior as long as the uncertain parameters and the disturbances are bounded. The robust control problem consists in the synthesis of the controller that will satisfy the system specifications and reject the given disturbances (Gutman 2001). Robustness is assured through a compromise of the performance criterion (Gendron *et al.* 1993). (Brdys *et al.* 2007) proposed a multilayered control structure that uses multiple time samples of the plant dynamics in order to achieve an optimal robust control of the wastewater treatment process.

Thus, multiple methods for the robust control were developed: starting with the classical control theory – QFT (Quantitative Feedback Theory) (Barbu *et al.* 2004; Caraman *et al.* 2005; Barbu *et al.* 2010), up to the modern control theory – H_∞ (Georgieva and Feyo de Azevedo 1999), H_2 (Halvarsson 2007) and μ (Sánchez-Peña and Sznaier 1998). (Georgieva and Feyo de Azevedo 1999) designed a H_∞ robust control structure for setpoint tracking starting from a simplified linear model and having as objective the disturbance rejection in the presence of noise and parameter variations of the process.

The main objective of this paper is to synthesize a H_∞ robust controller for a biological wastewater treatment process, in the presence of bounded variations of parametric uncertainties, disturbances and measurement noise. The paper is structured as follows: the first section describes the state of the art, the second section presents the nonlinear model of the wastewater treatment process, the third section is focused on the synthesis of the H_∞ robust controller and the paper ends with conclusions.

2. MATHEMATICAL MODEL OF THE BIOLOGICAL WASTEWATER TREATMENT PROCESS

Fig. 1 presents a conventional biological wastewater treatment process. The organic matter decomposition is achieved through biological oxidation using the populations of microorganisms developed in the aeration tanks. The wastewater treatment process is based on the maintenance of the sludge in suspension through pneumatic or mechanical aeration of the mixture. Aside from the biomass, represented by bacteria and other superior microorganism, the suspended solids also contain organic and inorganic particles. A percentage of the organic particles can be decomposed by hydrolysis, while the unaffected remained particles represent the inert mass. Thus, the substrate is removed from the wastewater and more biomass is produced. The suspended material is separated from the treated water by adding a secondary clarifier, from which the biomass is extracted in continuous mode. A part of the settled sludge is recycled, while the excedentary sludge is removed from the treatment plant. The effluent is continuously collected from the superior section of the clarifier and it is sent for further processing or it is discharged into a natural receiver if it meets the proper legal conditions.

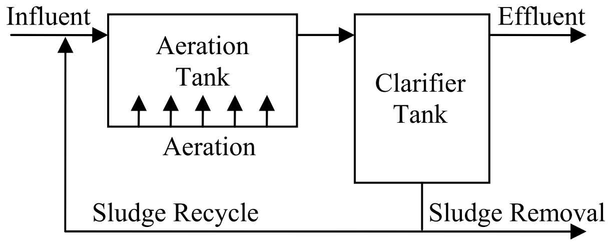

Fig. 1. Activated Sludge Process.

The mathematical model of the biological wastewater treatment process considered in this paper is a modified version of the 4th order Nejjari model (Ifrim 2012) described by the following equations:

$$\frac{dX}{dt} = \mu(t)X(t) - \mu_S X(t) + rD(t)X_r(t) - D(t)X(t) - rD(t)X(t) \quad (1)$$

$$\frac{dS}{dt} = -\frac{1}{Y}\left(\mu(t)X(t) - \mu_S X(t)\right) + D(t)\left(S_{IN} - S(t)\right) - rD(t)S(t) \quad (2)$$

$$\frac{dDO}{dt} = -\frac{1-Y}{Y}\left(\mu(t)X(t) - \mu_S X(t)\right)\cdot 10^3 + \alpha W \cdot 60 \cdot \\ \cdot\left(DO_{sat} - DO(t)\right) + D(t)\left(DO_{in} - DO(t)\right) - rD(t)DO(t) \quad (3)$$

$$\frac{dX_r}{dt} = D_S X(t) + rD_S\left(X(t) - X_r(t)\right) - \beta D_S X_r(t) - \\ - D_S(1-\beta)\cdot\eta\cdot X_r(t) \quad (4)$$

$$\mu(t) = \mu_{\max}\frac{S(t)}{K_S + S(t)}\frac{DO(t)}{K_{DO} + DO(t)} \quad (5)$$

where: $X(t)$ – biomass (the sludge); $S(t)$ – substrate (organic substance concentration); $DO(t)$ – dissolved oxygen concentration; $X_r(t)$ – recycled sludge; $D(t)$ – dilution rate; D_S – dilution of sludge compartment; S_{in} and DO_{in} – substrate and dissolved oxygen concentrations in the influent; Y – biomass yield factor; μ – biomass growth rate; μ_S – decay coefficient for biomass; μ_{MAX} – maximum specific growth rate; K_S and K_{DO} – saturation constants; DO_{sat} – dissolved oxygen saturation concentration; α – oxygen transfer rate; W – aeration rate; r and β – ratio of recycled and waste flow of the influent, η– parameter of the clarifier model.

The model has been identified on experimental data that were collected from a biological wastewater treatment pilot plant, presented in Fig. 2 and installed at University "Dunarea de Jos", Galati. The values of the parameters are presented in Table 1.

Fig. 2. The biological wastewater treatment pilot plant, from "Dunarea de Jos" University of Galati.

Table 1. Aerobic growth model parameters.

Group	Parameter	Value	Typical values	Unit
Kinetic parameters	μ_{max}	0.11	0.12–0.55	h^{-1}
	μ_S	0.02	0.002–0.07	h^{-1}
	K_S	0.18	0.01–0.18	$g{\cdot}L^{-1}$
	K_{DO}	0.20	0.01–0.5	$mg{\cdot}L^{-1}$
Stoichiometric parameters	Y	0.67	0.46–0.69	–
Transfer parameters	α	0.0033		–
	DO_{sat}	8.00		$mg{\cdot}L^{-1}$
	r	1.00		–
	β	0.20		–
Plant geometry	V	35.00		L
	V_S	6.00		L
Settling parameters	η	0.80		–

The initial conditions considered in the simulation are:

$X(0) = 0.7\ g{\cdot}L^{-1}$, $S(0) = 1.2\ g{\cdot}L^{-1}$, $DO(0) = 2\ mg{\cdot}L^{-1}$, $X_r(0) = 0.7\ g{\cdot}L^{-1}$, $S_{IN} = 1.2\ g{\cdot}L^{-1}$; $DO_{IN} = 2\ mg{\cdot}L^{-1}$; $D = 0.0172\ h^{-1}$; $W = 4.804\ L{\cdot}min^{-1}$.

The evolutions of the main variables of the open loop wastewater treatment process are presented in Fig. 3. In this model, the substrate concentration in the effluent reaches the $0.1\ g{\cdot}L^{-1}$ value, while the standard limit established by law is below $0.125\ g{\cdot}L^{-1}$.

A manner to achieve the organic substrate concentration in the effluent as required by the law is to directly control the organic substrate concentration through the dilution rate. The measurable output S, expressed through the COD (Chemical Oxygen Demand) value, can be measured online by means of a specific analyzer or estimated through an intermediate state estimator proposed by (Barbu 2009). The main disturbance of the wastewater treatment process is produced by the organic substrate concentration from the influent S_{in} (influent load of the wastewater treatment plant) An implicit disturbance of the system is the input flow F_{in} that depends directly on the pluviometric regime and that can be determined by using a flowmeter. The organic substrate control is associated with an appropriate control of the dissolved oxygen concentration (Ifrim 2012), where DO can be measured by optical or electrochemical sensors and it is controlled through the aeration rate. The proposed system consists in two inputs $u = [D \quad W]^T$, four states $x = [X \quad S \quad DO \quad X_r]^T$ and two measurable output variables $y = [S \quad DO]^T$.

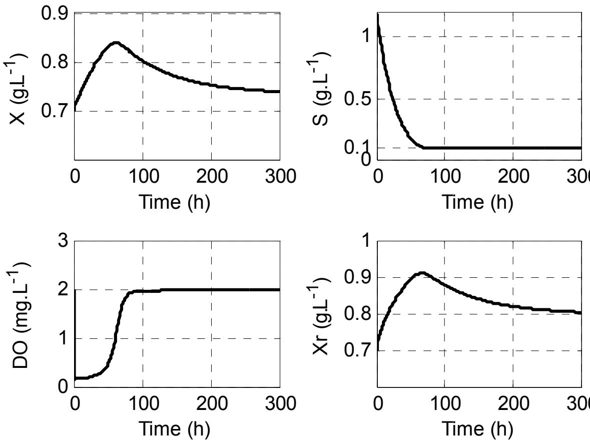

Fig. 3. The simulation results of the open loop model.

Romania's climate is temperate-continental of transition, with oceanic influences from the West, Mediterranean ones from South-West and continental-excessive ones from the East.

The measurement of water quantities coming from atmospheric precipitations or deposited by other hydrometeors is carried out by means of pluviometer and the continuous recording of precipitations (liquids) is performed with the pluviograph. Water quantities are daily measured at climatic deadlines (1, 7, 13, 19 local average solar time),) and they are expressed by the thickness of the fallen water layer, in mm (1mm=1 l/m²).

Yearly precipitations decrease in intensity from west to east, from over 600 mm to less 500 mm in the East Romanian Plain, under 450 mm in Dobrogea and about 350 mm by seaside, in the mountainous areas they reach 1000-1500 mm.

Romanian Statistical Yearbook

Fig. 4 emphasizes graphically the pluviometrical regime between the years 1901-2000, using statistical data furnished by the Romanian Statistical Yearbook in which it can be found the monthly distribution of the average precipitation values for a sum of weather stations across the country. Analyzing this regime, the precipitation abundance of the warm season can be observed, while the cold season reaches lower precipitation quantities. Thus, in opposition with the months associated with winter, that are low in precipitations, the most abundant in precipitation months are May, June and July. As a conclusion, the pluviometrical regime is characterized by maximum values in summer and minimum values in winter. Speaking in percents, both the ascending and the descending trends do not exceed 33.33%. Applying these observations to the influent organic load (6), the wastewater treatment process is controlled considering the influent flow for one year interval:

$$S_{in} = \frac{S_N \cdot F_N}{F_N + F_P - F_S} \tag{6}$$

where: S_N – organic substrate concentration for normal conditions, F_N – influent flow for normal conditions, F_P – influent flow for rain conditions, F_S – influent flow for dryness conditions. Also, a 1% daily variation of the domestic wastewater was taken into consideration.

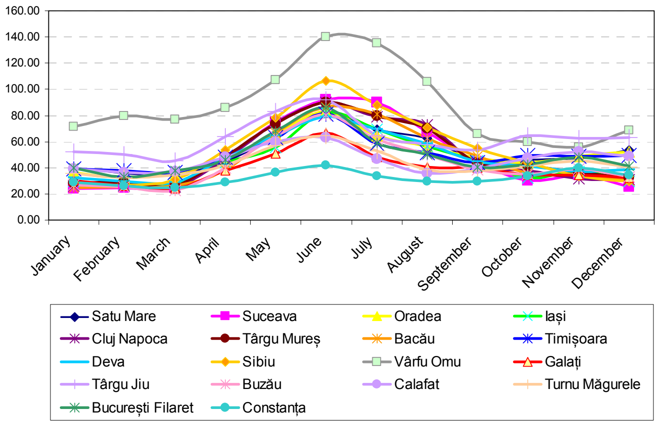

Fig. 4. Monthly average precipitation values between the years 1901 – 2000.

3. SYNTHESIS OF H_∞ CONTROLLER

The main objective of the biological wastewater treatment process is to obtain a concentration of organic substrate, in the effluent, that is below the value imposed by the law. A high number of wastewater treatment plants operate continuously, considering the dilution rate as the main control input that has a significant influence over the process.

The substrate concentration is directly affected by the aerobic growth of the biomass (activated sludge) and indirectly by the dynamics of the dissolved oxygen concentration. Although the biological wastewater treatment process can present significant interferences between the control channels, it can also be analyzed as being decoupled, by means of two input/output pairs: 1. the dilution rate – the organic substrate concentration, and 2. the aeration rate – the dissolved oxygen concentration. Thus, following the six steps needed by the synthesis (Georgieva and Feyo de Azevedo 1999), a H_∞ controller has been designed for each channel pair.

P1. Mathematical modeling and linearization of the nonlinear model

Besides the nominal system a proper robust controller also stabilizes the associated nearby class of systems.

To apply a robust control strategy, the nonlinear characteristic of the activated sludge wastewater treatment process, requires the linearization of the considered model and the development of a linear model with uncertain parameters. The sources of these uncertainties are summed up to be the sludge dynamics and the influent organic substrate concentration. The daily period variation of the influent is the main disturbance of the process, followed by the monthly sewage precipitation data.

The transfer functions of the dilution rate – organic substrate concentration channel (7) and the aeration rate – dissolved oxygen concentration channel (8) are obtained after linearization using *linmod* function of MATLAB. From the transfer function (7), respectively (8), the transfer function (9), respectively (10) will result, through simplification that follows the frequency analysis.

$$P_{SD}(s) = \frac{s^3 + 1.7968s^2 + 0.3485s + 0.0046}{s^4 + 2.0633s^3 + 0.6933s^2 + 0.0680s + 0.0008} \tag{7}$$

$$P_{DOW}(s) = \frac{1.1887s^3 + 0.5815s^2 + 0.0738s + 0.0009}{s^4 + 2.0633s^3 + 0.6933s^2 + 0.0680s + 0.0008} \tag{8}$$

$$P_{SD}(s) = \frac{s + 1.5780}{s^2 + 1.8460s + 0.2898} \tag{9}$$

$$P_{DOW}(s) = \frac{1.1890 \cdot s + 0.3189}{s^2 + 1.8460s + 0.2898} \tag{10}$$

Fig. 5 presents the general closed-loop structure, where: *P(s)* – the wastewater biological treatment process, *K(s)* – the controller, *M(s)* – the measurement sensor, $W_{\check{S}}$ – the sensitivity weight function, $W_{\check{T}}$ – the complementary sensitivity weight function, *S/DO* – the organic substrate and the dissolved oxygen concentrations, *S*/DO** – the setpoints of the organic substrate and dissolved oxygen concentrations, S_m/DO_m – the measured organic substrate and dissolved oxygen concentrations, *e(s)* – the error, *D* – the dilution rate, *W* – the aeration rate, *n* – the measurement noise, *d* – the system disturbances, *i* – the parametric uncertainties, z_1 and z_2 – the controlled components.

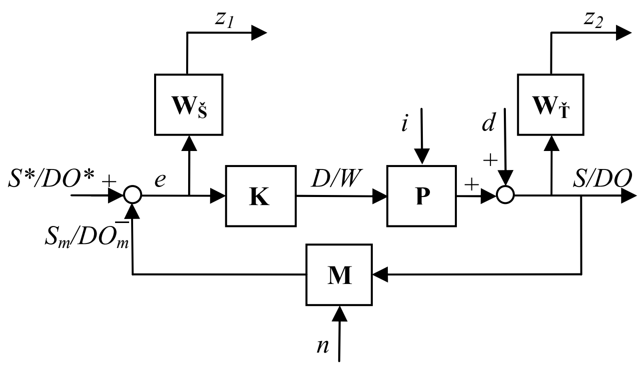

Fig. 5. Feedback control structure.

The controller must assure the tracking of the setpoints *S** and *DO** having low errors and compensating the external disturbances effect and the measurement noise, where the disturbances S_{in} and F_{in} are considered to be measurable. In this paper, *S* identifies the organic substrate concentration and *DO* identifies the dissolved oxygen concentration, as controlled variables.

The kinetic parameter uncertainties and the operating conditions were considered to have similar characteristics with the domestic wastewater. The model uncertainties effect embeds sinusoidal variations of both signals and of the parameters that are presented in Table 2.

Table 2. Sinusoidal variation of the wastewater treatment process variables.

Sinusoidal variation	MIN value	NOM value	MAX value
$\mu_{\max} = \mu_{\max}^{nom} + 0.22 \cdot \sin\left(\frac{\pi}{12} t\right)$	0.11	0.33	0.55
$K_S = K_S^{nom} + 0.085 \cdot \sin\left(\frac{\pi}{12} t + \frac{3\pi}{12}\right)$	0.01	0.095	0.18
$Y = \mu_{\max}^{nom} + 0.115 \cdot \sin\left(\frac{\pi}{12} t\right)$	0.46	0.575	0.69
$r = r^{nom} + 1.7 \cdot \sin\left(\frac{\pi}{12} t\right)$	0.3	2.00	3.7
$S_{in} = S_{in}^{nom}\left(1 + 0.33 \cdot \sin\left(\frac{\pi}{12} t\right)\right)$	$0.66S_{in}$	1.20	$0.66S_{in}$
$F_{in} = \left(F_{in}^{nom}\left(1 + 0.33 \cdot \sin\left(\frac{\pi}{3648} t + \frac{2.82\pi}{2}\right)\right)\right) \cdot \left(1 + 0.01 \sin\left(\frac{\pi}{12}\right)\right)$	$0.66F_{in}$	1.20	$0.66F_{in}$
$S_{ref} = S_{ref}^{nom} + 0.0415 \cdot \sin\left(\frac{\pi}{12} t\right)$	0.04	0.084	0.125
$DO_{ref} = DO_{ref}^{nom} + \sin\left(\frac{\pi}{12} t\right)$	1	2	3

The closed-loop system outputs (11), (12) and the tracking errors (13), (14) impose the minimization of the sensitivity functions $\breve{S}_S(s)$ and $\breve{S}_{DO}(s)$, in order to reduce the effects of disturbances and errors, and of the complementary sensitivity functions, $\breve{T}_S(s)$ and $\breve{T}_{DO}(s)$, in order to attenuate the measurement noise effect.

$$S(s) = \frac{1}{1 + K_S(s) \cdot P(s) \cdot P_m(s)} \cdot \left(P(s) \cdot K_S(s) \cdot S^*(s) + d(s) - P(s) \cdot K_S(s) \cdot n(s)\right) \tag{11}$$

$$DO(s) = \frac{1}{1 + K_{DO}(s) \cdot P(s) \cdot P_m(s)} \cdot \left(P(s) \cdot K_{DO}(s) \cdot DO^*(s) + d(s) - P(s) \cdot K_{DO}(s) \cdot n(s)\right) \tag{12}$$

$$e_1(s) = S^*(s) - S_m(s) = \breve{S}_S(s) \cdot S^*(s) - \breve{S}_S(s) \cdot d(s) + \breve{T}_S(s) \cdot n(s) \tag{13}$$

$$e_2(s) = DO^*(s) - DO_m(s) = \breve{S}_{DO}(s) \cdot DO^*(s) - \breve{S}_{DO}(s) \cdot d(s) + \breve{T}_{DO}(s) \cdot n(s) \tag{14}$$

where:

$$\breve{S}_S(s) = \frac{1}{1 + K_S(s) \cdot P(s) \cdot P_m(s)}$$

and

$$\breve{S}_{DO}(s) = \frac{1}{1 + K_{DO}(s) \cdot P(s) \cdot P_m(s)}$$

represent the sensitivity functions, and

$$\breve{T}_S(s) = \frac{K_S(s) \cdot P(s)}{1 + K_S(s) \cdot P(s) \cdot P_m(s)}$$

and

$$\breve{T}_{DO}(s) = \frac{K_{DO}(s) \cdot P(s)}{1 + K_{DO}(s) \cdot P(s) \cdot P_m(s)}$$

represent the complementary sensitivity functions.

The goal of meeting the robustness and performance requirements is dependant on choosing the proper weighting functions for the wastewater treatment process.

P2. Performance description weighting functions based

The disturbance effect on the output signals of the system is characterized by the frequency behavior in closed-loop of the sensitivity functions $\breve{S}_S(s)$ and $\breve{S}_{DO}(s)$, given by (15) and (16). By minimizing $\breve{S}_S(s)$ and $\breve{S}_{DO}(s)$ sensitivity functions, the disturbance effect on the output signal and the tracking error are diminished. The disturbance and the weighting functions $W_{\breve{S}-S}(s)$ and $W_{\breve{S}-DO}(s)$ should be directly proportional. A necessary condition to be satisfied is that the sensitivity functions $\breve{S}_S(s)$ and $\breve{S}_{DO}(s)$ should be maintained below the inverse weighting functions $W_{\breve{S}-S}^{-1}(s)$ and $W_{\breve{S}-DO}^{-1}(s)$ in the low frequency band (17) and (18).

$$\breve{S}_S = \frac{1}{1 + P \cdot K_S} \tag{15}$$

$$\breve{S}_{DO} = \frac{1}{1 + P \cdot K_{DO}} \tag{16}$$

$$\left|\breve{S}_S\right| \le \left|W_{\breve{S}-S}\right|^{-1} \Leftrightarrow \left\|W_{\breve{S}-S} \breve{S}_S\right\|_\infty \le 1 \tag{17}$$

$$\left|\breve{S}_{DO}\right| \le \left|W_{\breve{S}-DO}\right|^{-1} \Leftrightarrow \left\|W_{\breve{S}-DO} \breve{S}_{DO}\right\|_\infty \le 1 \tag{18}$$

The closed-loop system robustness in the presence of disturbances is characterized by the frequency behavior, in closed-loop, of the complementary sensitivity functions $\breve{T}_S(s)$ and $\breve{T}_{DO}(s)$, given by (19) and (20). Minimizing the $\breve{T}_S(s)$ and $\breve{T}_{DO}(s)$ functions a higher degree of system robustness is achieved, along with the minimization of the measurement noise, thus reducing the closed-loop instability risk. A mandatory condition stipulates that the complementary sensitivity functions $\breve{T}_S(s)$ and $\breve{T}_{DO}(s)$ should be maintained below the inverse weighting functions $W_{\breve{T}-S}^{-1}(s)$ and $W_{\breve{T}-DO}^{-1}(s)$ in the high frequency band (21) and (22).

$$\breve{T}_S = \frac{P \cdot K_S}{1 + P \cdot K_S} \tag{19}$$

$$\breve{T}_{DO} = \frac{P \cdot K_{DO}}{1 + P \cdot K_{DO}} \tag{20}$$

$$\left|\breve{T}_S\right| \le \left|W_{\breve{T}-S}\right|^{-1} \Leftrightarrow \left\|W_{\breve{T}-S} \breve{T}_S\right\|_\infty \le 1 \tag{21}$$

$$\left|\breve{T}_{DO}\right| \le \left|W_{\breve{T}-DO}\right|^{-1} \Leftrightarrow \left\|W_{\breve{T}-DO}\breve{T}_{DO}\right\|_{\infty} \le 1 \quad (22)$$

Identifying the multiplicative uncertainty norm $\left|\Delta_m\right| \le w(s)$ precedes the identification of the highest uncertainty that is expected. Choosing for the weighting functions $W_{\breve{T}\text{-}S}(s)$ and $W_{\breve{T}\text{-}DO}(s)$ depends on the system transfer property of being proper or strictly proper, being necessary for the second case of proper structure $W_{\breve{T}\text{-}S}(s)P(s)$ and $W_{\breve{T}\text{-}DO}(s)P(s)$. Additionally, the crossover frequencies $W_{\breve{S}\text{-}S}(j\omega)$ and $W_{\breve{S}\text{-}DO}(j\omega)$, in Bode diagram, must precede the crossover frequencies $W_{\breve{T}\text{-}S}(j\omega)$ and $W_{\breve{T}\text{-}DO}(j\omega)$ aiming to fulfill the performance criteria.

In an attempt to avoid conflicts in regard to the unitary sum of the $\breve{S}_S(s)$ and $\breve{T}_S(s)$, respectively $\breve{S}_{DO}(s)$ and $\breve{T}_{DO}(s)$ functions, the weighting functions can be defined for different frequency intervals (Georgieva and Ignatova 1999): $\breve{S}_S(s)$, $\breve{S}_{DO}(s)$ must be low in the low frequency band in order to reduce the additive disturbance effect on the output signal, and $\breve{T}_S(s)$, $\breve{T}_{DO}(s)$ must be low in the high frequency band in order to reduce the measurement noise effect on the output signal.

Moreover, the additional weighting functions $W_{\breve{S}\text{-}S}(s)$, $W_{\breve{S}\text{-}DO}(s)$ and $W_{\breve{T}\text{-}S}(s)$, $W_{\breve{T}\text{-}DO}(s)$ ensure flexibility to the performance specifications. In order to synthesize a controller that would compensate uncertainties, the cost function $\|T_{zw}\|_{\infty}$ must be lower than a particularly constant γ. A necessary and sufficient condition to achieve robust performance is stated in the inequalities (23) and (24).

$$\left\|\left|W_{\breve{S}-S}\breve{S}_S\right| + \left|W_{\breve{T}-S}\breve{T}_S\right|\right\|_{\infty} \le 1 \quad (23)$$

$$\left\|\left|W_{\breve{S}-DO}\breve{S}_{DO}\right| + \left|W_{\breve{T}-DO}\breve{T}_{DO}\right|\right\|_{\infty} \le 1 \quad (24)$$

The daily period of the wastewater treatment process with activated sludge represents a condition in choosing a high weight nearby to $\pi/12\ h^{-1}$ as a constraint for the sensitivity functions $\breve{S}_S(s)$ and $\breve{S}_{DO}(s)$, to compensate the variations for a period higher than $2h$, or the equivalent of choosing a $\pi\ h^{-1}$ bandwidth (Georgieva and Feyo de Azevedo 1999), thus being chosen the weighting functions stated in (25), (26) and (27), (28).

$$W_{\breve{S}-S}^{-1} = 0.01 \cdot \frac{150 \cdot s + 1}{0.3 \cdot s + 1} \quad (25)$$

$$W_{\breve{T}-S}^{-1} = \frac{0.01 \cdot s^2 + 0.1 \cdot s + 1}{0.1 \cdot s^2 + 0.01 \cdot s + 0.01} \quad (26)$$

$$W_{\breve{S}-DO}^{-1} = 0.01 \cdot \frac{550 \cdot s + 1}{2.3 \cdot s + 1} (26) \quad (27)$$

$$W_{\breve{T}-DO}^{-1} = \frac{0.01 \cdot s^2 + 0.1 \cdot s + 1}{0.1 \cdot s^2 + 0.01 \cdot s + 0.01} \quad (28)$$

Choosing $W_{\breve{S}-S}^{-1}(0) = W_{\breve{S}-DO}^{-1}(0) = 0.01$ has the significance of a maximum steady-state tracking error of 1%, while $W_{\breve{S}-S}^{-1}(s \to \infty) = 5$ and $W_{\breve{S}-DO}^{-1}(s \to \infty) = 2.39$ signify a high frequency disturbance amplification of 5 and 2.39, respectively.

Considering the crossover frequencies constraint and the selection of $W_{\breve{T}\text{-}S}(s)$ and $W_{\breve{T}\text{-}DO}(s)$ so that they will correspond to the desired 2π *(24h)* bandwidth, it is required for the complementary sensitivity functions to be low nearby the frequency where the measurement noise is amplified.

P3. Two-port state-space representation of the augmented process model

The extended multivariable process, outlined in Fig. 6 and Fig. 7, is based on the monovariable linearized model, to which the weighting functions are added.

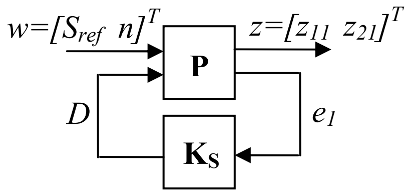

Fig. 6. Linear Fractional Transformation model for the dilution rate – organic substrate concentration control pair.

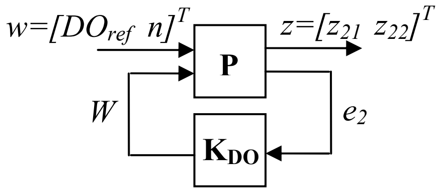

Fig. 7. Linear Fractional Transformation model for the aeration rate – dissolved oxygen concentration control pair.

The closed-loop transfer functions between the controlled outputs z and the exogenous inputs w, are given by the equations (29) and (30), for both cases:

$$T_{zw-S} = P_{11} + P_{12} \cdot K_S \cdot (I - P_{22}K_S)^{-1} \cdot P_{21} \quad (29)$$

$$T_{zw-DO} = P_{11} + P_{12} \cdot K_{DO} \cdot (I - P_{22}K_{DO})^{-1} \cdot P_{21} \quad (30)$$

where: $T_{zw\text{-}S}$ and $T_{zw\text{-}DO}$ are called Linear Fractional Transformations (LFT) and have the state space description in (31) – (41):

$$A_S = \begin{bmatrix} -1.846 & -0.2898 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ -1 & -1.578 & -0.0067 & 0 & 0 \\ 1 & 1.578 & 0 & -10 & -100 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \quad (31)$$

$$B_{S1} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}; \; B_{S2} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (32)$$

$$C_{S1} = \begin{bmatrix} -0.2 & -0.3156 & 0.6653 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 10 & 15.78 & 0 & -99 & -990 \end{bmatrix} \quad (33)$$

$$C_{S2} = \begin{bmatrix} -1 & -1.578 & 0 & 0 & 0 \end{bmatrix} \quad (34)$$

$$D_{S11}=\begin{bmatrix}0.2\\0\\0\end{bmatrix};\; D_{S12}=\begin{bmatrix}0\\1\\0\end{bmatrix};\; D_{S21}=1;\; D_{S22}=0 \quad (35)$$

$$A_{DO}=\begin{bmatrix}-1.846 & -0.2898 & 0 & 0 & 0\\ 1 & 0 & 0 & 0 & 0\\ -1.189 & -0.3189 & -0.0018 & 0 & 0\\ 1.189 & 0.3189 & 0 & -10 & -100\\ 0 & 0 & 0 & 1 & 0\end{bmatrix} \quad (36)$$

$$B_{DO1}=\begin{bmatrix}0\\0\\1\\0\\0\end{bmatrix};\; B_{DO2}=\begin{bmatrix}1\\0\\0\\0\\0\end{bmatrix} \quad (37)$$

$$C_{DO1}=\begin{bmatrix}-0.4972 & -0.1334 & 0.1811 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\\ 11.89 & 3.189 & 0 & -99 & -999\end{bmatrix} \quad (38)$$

$$C_{DO2}=\begin{bmatrix}-1.189 & -0.3189 & 0 & 0 & 0\end{bmatrix} \quad (39)$$

$$D_{DO11}=\begin{bmatrix}0.4182\\0\\0\end{bmatrix};\; D_{DO12}=\begin{bmatrix}0\\1\\0\end{bmatrix};\; D_{DO21}=1 \quad (40)$$

$$D_{DO22}=0 \quad (41)$$

All the initial assumptions regarding the existence of a solution for the H_∞ control problem are satisfied:

A1. $D_{11}=0$;

A2. The pairs (A_S, B_{S2}) and (A_{DO}, B_{DO2}) are stabilizable;

A3. The pairs (C_{S2}, A_S) and (C_{DO2}, A_{DO}) are detectable;

A4. The matrix rank of D_{12} is equal to the dimension of the control input u, hence the $rankD_{S12}=1$ and the $rankD_{DO12}=1$;

A5. The matrix rank of D_{21} is equal to the dimension of the output y, hence the $rankD_{S21}=1$ and the $rankD_{DO21}=1$.

The A2 and A3 assumptions are necessary in order to guarantee the existence of a controller and A4 and A5 assumptions are necessary in order to guarantee that the controller transfer function is proper.

P4. State-space H_∞ control design

Based on the extended models (31-35 and 36-41) of the wastewater treatment process, the transfer functions for the H_∞ stable controllers (42) and (43) have the same number of states as the augmented plant.

$$K_S=\frac{2.57s^4+30.45s^3+305.33s^2+482.10s+74.51}{s^5+19.27s^4+178.19s^3+605.63s^2+587.95s+3.89} \quad (42)$$

$$K_{DO}=\frac{47.9s^4+567.7s^3+5691.1s^2+8985.9s+1388.9}{s^5+70.2s^4+9777.4s^3+6296.4s^2+3603.8s+6.5} \quad (43)$$

The closed loop system poles of the dilution rate – organic substrate concentration control pair are:

$$\begin{Bmatrix}-7.0575\ \pm 7.0588i;-5\ \pm\ 8.6603i;-2.4714;-1.7216;-1.6728;\\ -0.9663;-0.1732;-0.0067\end{Bmatrix}$$

and the closed loop system poles of the aeration rate – dissolved oxygen concentration control pair are:

$$\begin{Bmatrix}-53.0277;-7.6890\pm 6.3381i;-5.0000\pm 8.6603i;-1.6728;\\ -1.5580;-0.1871;-0.1732;-0.0018\end{Bmatrix}$$

P5. Frequency domain robust analysis

Figures 8 and 10 present the sensitivity function and the complementary sensitivity function, in respect to the fulfillment of the performance requirements (17) – (18) and (21) – (22) as well as the crossover frequency of $W_{\check{S}\text{-}S}(s)$ and $W_{\check{S}\text{-}DO}(s)$ preceding the crossover frequency of $W_{\check{T}\text{-}S}(s)$ and $W_{\check{T}\text{-}DO}(s)$. Figures 9 and 11 show the characteristics of the cost functions $T_{zw\text{-}S}$ and $T_{zw\text{-}DO}$ in terms of which the scaling of the weight functions has been performed.

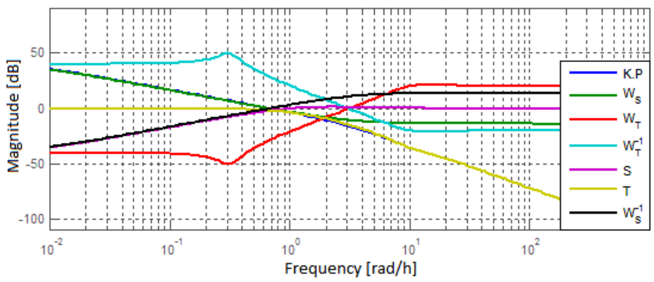

Fig. 8. Bode characteristics for the dilution rate – organic substrate concentration control channel.

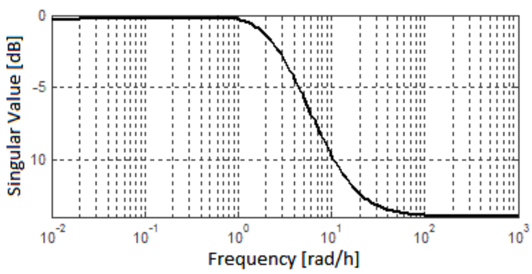

Fig. 9. Cost function characteristics for the dilution rate – organic substrate concentration control channel.

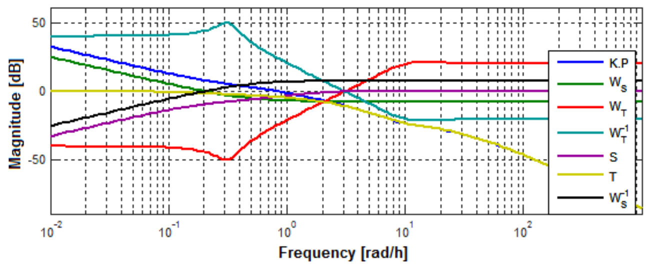

Fig. 10. Bode characteristics for the aeration rate – dissolved oxygen concentration control channel.

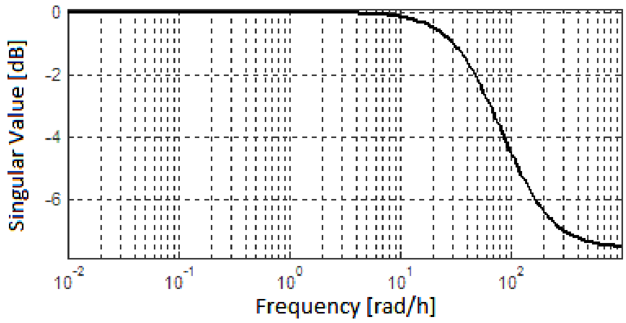

Fig. 11. Cost function characteristics for the aeration rate – dissoved oxygen concentration control channel.

P6. Simulation of the H_∞ *controlled nonlinear process*

Fig. 12 presents the general scheme of the multivariable control of biological wastewater treatment process with activated sludge, having as controlled outputs the organic substrate and the dissolved oxygen concentrations.

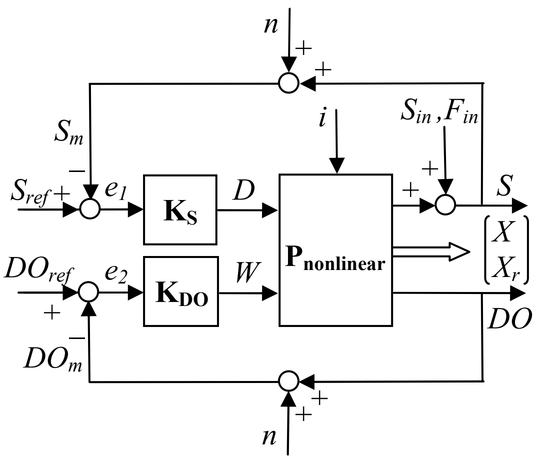

Fig. 12. Multivariable control scheme of the wastewater treatment process.

In Fig. 13, the three zoomed sections contain the daily variation during low precipitation conditions (0 – 48 *h* corresponding to the 1st and 2nd of January), during high precipitation conditions (3984 – 4008 *h* corresponding to the 15th of June) and during normal precipitation conditions (5856 – 5880 *h* corresponding to the 1st of September). The results obtained through simulation are shown in Fig. 14 – Fig. 19.

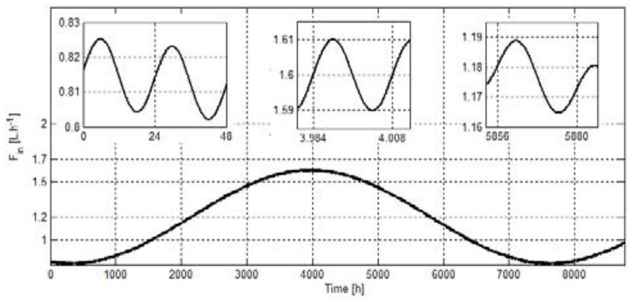

Fig. 13. Daily sinusoidal variation of the precipitations over a year.

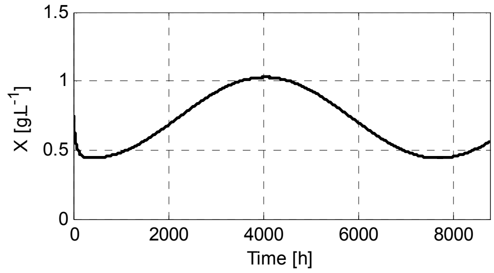

Fig. 14. The evolution of the biomass concentration.

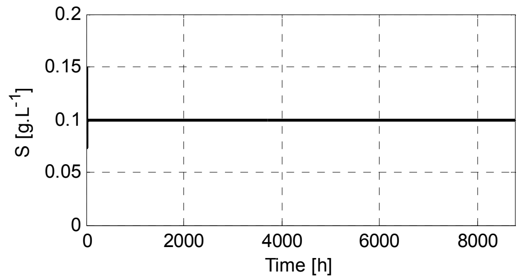

Fig. 15. The evolution of the organic substrate concentration.

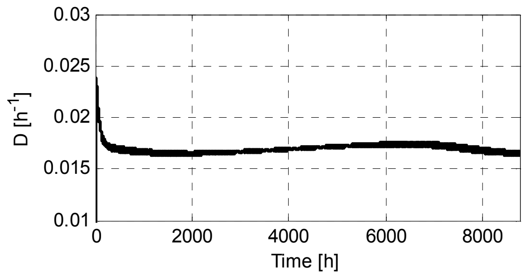

Fig. 16. The evolution of the dilution rate.

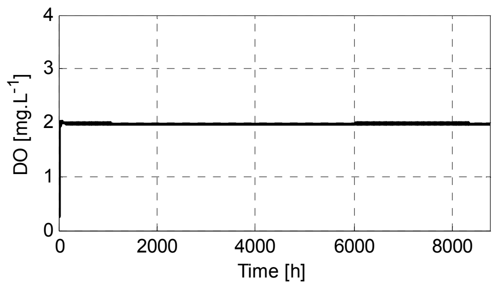

Fig. 17. The evolution of the dissolved oxygen concentration.

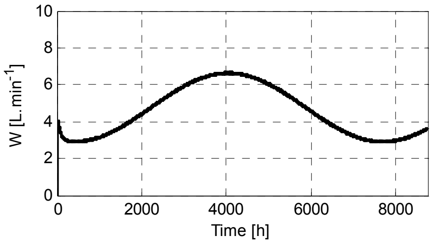

Fig. 18. The evolution of the aeration rate.

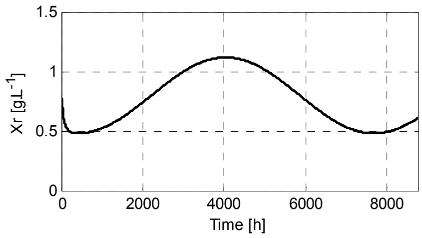

Fig. 19. The evolution of the recycling biomass concentration

In order to validate the performance requirements of the nonlinear multivariable control system, the simulations were performed in respect to the bounded sinusoidal variations presented in Table 1. The simulation results are embedded in Fig. 20 – Fig. 23.

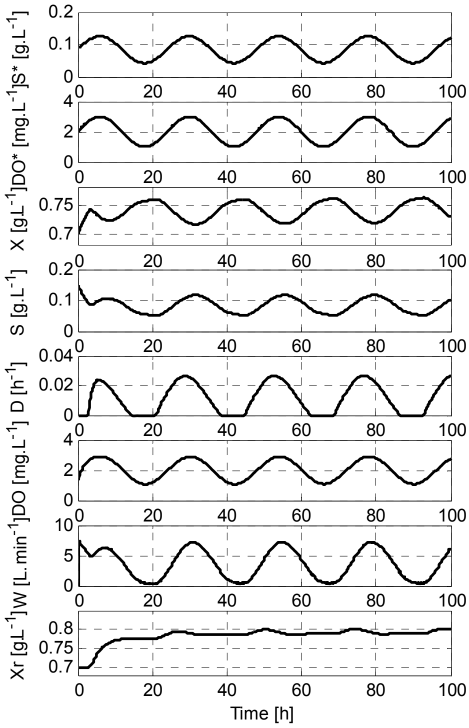

Fig. 20. The simulation results for the sinusoidal setpoints of S^* and DO^*.

Fig. 20 embeds the sinusoidal representation of the organic substrate and the dissolved oxygen concentrations setpoints, whereas Fig. 21 – Fig. 23 embed the step representation of the same setpoints, $S^* = 0.1 g \cdot L^{-1}$ and $DO^* = 2 mg \cdot L^{-1}$. Fig. 21 presents the system's evolution in the presence of the sinusoidal variation of the μ_{max} parameter, while Fig. 22 presents the system's evolution in the presence of the sinusoidal disturbance S_{in}. Fig. 23 considers the evolution of the controlled wastewater treatment process in the presence of the sinusoidal variation of the parameters μ_{max}, K_S, Y, the recycling rate r, the disturbance S_{in} and the 30% variation of the measurement noise. All the simulation results confirm a good behavior in tracking the organic substrate concentration and the dissolved oxygen concentration setpoints, considering the worst case scenario for the sinusoidal form of the variations of the parametric uncertainties, the load disturbances and the measurement noise.

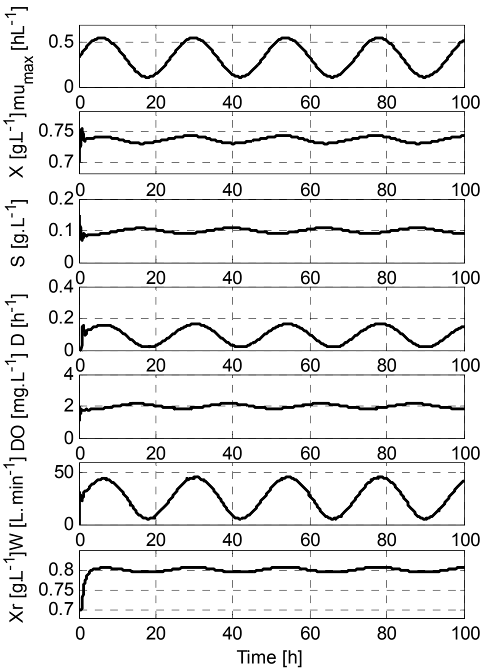

Fig. 21. The simulation results in the presence of the sinusoidal variation of the parameter μ_{max} .

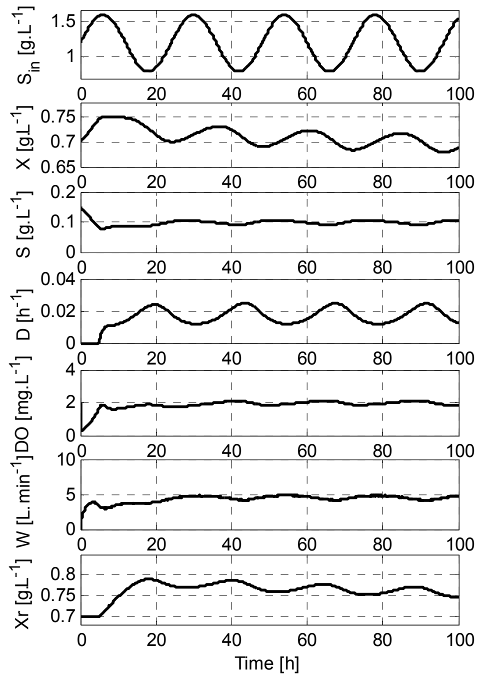

Fig. 22. The simulation results in the presence of the sinusoidal variation of the disturbance S_{in}.

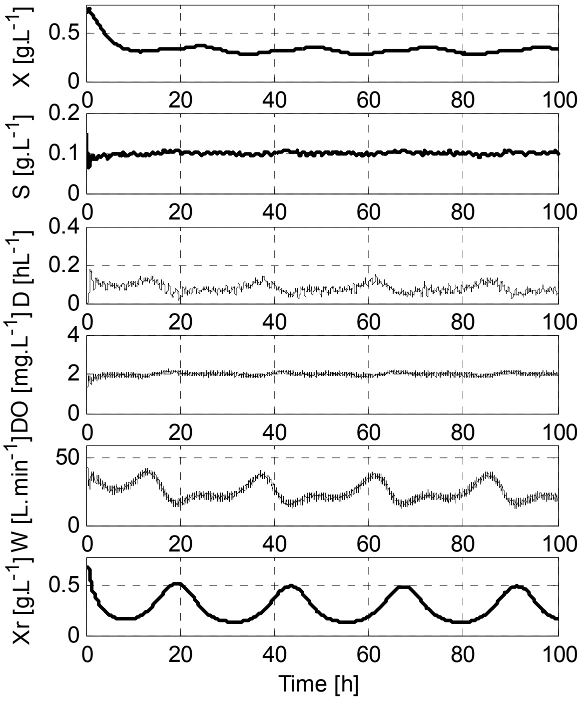

Fig. 23. The simulation results in the presence of the sinusoidal variation of the parameters μ_{max}, K_S, Y, the recycling rate r, the disturbance S_{in} and the 30% variation of the measurement noise.

4. CONCLUSIONS

This paper deals with the analysis of robust mixed sensitivity tracking error of the closed-loop multivariable system using the sinusoidal form model for representing the uncertain parameters of the model. The robust multivariable control has been analyzed and performed considering the following control pairs: dilution rate – organic substrate concentration and the aeration rate – dissolved oxygen concentration, where the organic substrate concentration in the influent S_{in} is the main process disturbance that has a daily sinusoidal variation. The kinetic parameter uncertainties and the operating conditions were selected to reflect the characteristics of the typical domestic wastewater.

The robust multivariable control problem contains, additionally to the previous cases, an analysis of the system under the influence of the precipitation gauge F_{in} simulated during an entire year timeframe, and sampled per monthly precipitation mean values.

The main role of the robust controller is to enforce the system in tracking the organic substrate concentration and the dissolved oxygen concentration setpoints, with small errors, in a time varying process, while compensating for the effect of external disturbances and measurement noise.

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REFERENCES

Ayesa, E., De la Sota, A., Grau, P., Sagarna, J.M., Salterain, A., and Suescun, J. (2006). Supervisory control strategies for the new WWTP of Galindo-Bilbao: the long run from the conceptual design to the full-scale experimental validation, *Water Science and Technology*, 53(4-5): 193-201.

Barbu, M., Caraman, S., and Ceanga, E. (2004). QFT Robust Control of Biotechnological Processes, *IEEE-TTTC International Conference on Automation, Quality and Testing, Robotics, AQTR 2004*, Cluj-Napoca, Proceesings Tome 1, 129-134.

Barbu, M. (2009). Conducerea automata a proceselor biotehnologice, *Galati University Press*, Galati, Romania.

Barbu, M., Ifrim, G., Caraman, S., Bahrim, G. (2010). QFT Control of Dissolved Oxygen Concentration in a Wastewater Treatment Pilot Plant, *In Proceedings of 11th IFAC Symposium on Computer Applications in Biotechnology*, Leuven, Belgium.

Brdys, M.A., and Zhang, Y. (2001). Robust Hierarchical Optimizing Control of Municipal Wastewater Treatment Plants, *Preprints of the 9th IFAC/IFORS/IMACS/IFIP Symposium Large Scale Systems: Theory & Applications – LSS'2001*, Bucharest, Romania, 540-547, July 18-20.

Brdys, M.A., Grochowski, M., Gminski, T., Konarczak, K., and Drewa, K. (2007). Hierarchical predictive control of integrated wastewater systems, *Control Engineering Practice*.

Buhr, H.O., Andrews, J.F., and Keinath, T.M. (1974). Research needs for automation of wastewater treatment systems, *In: Proceedings of a U.S. Environmental Protection Agency Workshop*, Clemson University, South Carolina, September 23-25, 135, 1974.

Buitron, G., Moreno, Y., Betancur, M., and Moreno, J. (2004). Application of the event-driven time optimal control strategy for the degradation of inhibitory wastewater in a discontinuous bioreactor, *Proceedings of the 4th IWA World Water Comgress*, Marrakech, 19-24 September.

Busby, J.B., and Andrews, J.F. (1975). Dynamic modeling and control strategies for the activated sludge process, *J. Water Poluut. Control Fed*., 47: 1055-1080.

Caraman, S., Barbu, M., and Ceanga, E. (2005). Robust multimodel control using QFT techniques of a wastewater treatment process, *Journal of CEAI*, 7(2): 10-17.

Copp, J.B., Spanjers, H., and Vanrolleghem, P.A. (2002)., (eds), Respirometry in control of the activated sludge process: benchmarking control strategies, *IWA STR No 11*, IWA Publishing, London, UK, 192.

Dircks, K., Beun, J.J., Van Loosdrecht, M.C.M., Heijnen, J.J., and Henze, M. (2001). Glycogen metabolism in aerobic mixed cultures. *Biotechnol. BioEng*., 73(2): 85–94.

Dochain, D., and Perrier, M. (1992). Adaptive linearizing control of the activated sludge process, *In: Proceedings Workshop Modelling, Monitoring and Control of the Activated Sludge Process*, Med. Fac. Landbouww, Rijksuniv. Gent, 57: 2239-2248.

Dold, P.L., Ekama, G.A., and Marais, V.v.R. (1980). A general model for the activated sludge process, *Prog. Wat. Tech.*, 12(6): 47-77.

Eckenfelder, W.W. Jr., and O'Conner, D.J. (1955). The Aerobic Treatment of Organic Wastes, *Proc. 9th Ind. Waste Conf.*, Purdue Univ. Lafayette, Ind. Ext. Serv. 87: 39.

Eckenfelder, W.W. Jr. (1966). Industrial Water Pollution Control, *McGraw-Hill*, New York.

Georgieva, P.G., and Feyo de Azevedo, S. (1999). Robust control design of an activated sludge process. *International Journal of Robust and Nonlinear Control*, 9: 949-967.

Georgieva, P.G., and Ignatova, M.N. (1999). LFT models of continuous biotechnological process, *J. Bioprocess Engng.*, 20(2): 179-183.

Gernaey, K.V., Loosdrecht, M.C.M., Henze, M., Lind, M., and Jørgensen, S.B. (2004). Activated sludge wastewater treatment plant modelling and simulation: state of the art, *Environmental Modelling & Software, Elsevier*, 19: 763-783.

Gendron, S., Perrier, M., Barrette, J., Amjad, M., Holko, A., and Legault, N. (1993). Deterministic adaptive control of SISO processes using model weighting adaptation, *Ind. J. Control.*

Goodman, B.L., and Englande, A.J. (1974). A Unified Model of the Activated Sludge Process. *Journal of Water Pollution Control Federation*, 46: 312-332, 1974.

Grigorieva, E.V., and Khailov, E.H. (2010). Optimal control of a waste water cleaning plant, *Eighth Mississippi State – UAB Conference on Differential Equations and Coputational Simulations*, Electronic Journal of Differential Equations, 19: 161-175.

Gujer, W., and Kappeler, J. (1992). Modelling Population Dynamics in Activated Sludge Systems, *Wat. Sci. Tech.*, 25(6), 93-104.

Gutman, P.O. (2001). *QSYN the toolbox for robust control systems design for use with MATLAB 6.0*.

Halvarsson, B. (2007). Interaction Analysis and Control of Bioreactors for Nitrogen Removal, *Dissertation Thesis, Uppsala University*, Sweden, 120.

Henze, M., Grady, C.P.L. Jr., Gujer, W., Marais, G.V.R., and Matsuo, T. (1987). Activated Sludge Model No.1, *IAWPRC Scientific and Technical Report No.1*, IAWQ, Great Britain.

Henze, M., Gujer, W., Mino, T., Matsuo, T., Wentzel, M.C., and Marais, G.V.R. (1995). Activated Sludge Model No.2, *IAWQ Scientific and Technical Report No.3*, IAWQ, Great Britain.

Henze, M., Gujer, W., Mino, T., and van Loosdrecht, M. (2000). Activated Sludge Models ASM1, ASM2, ASM2d and ASM3*, IWA Publishing*, London, Great Britain.

Jeppsson, U. (1996). Modelling aspects of wastewater treatment processes, *Ph.D. thesis, Dept. of Industrial Electrical Eng. And Automation*, Lund University, Sweden.

Ifrim, G.A. (2012). Control of Two Biological Processes of Environmental Interest (biological wastewater treatment and microalgae production in photobioreactor), *Ph.D thesis, "Dunarea de Jos" University*, Galati, 243.

Lawrence, J.W., and McCarty, P.L. (1970). Unified basis for biological treatment design and operation*, J. San. Engng. Div.* ASCE, 96: 757-777.

Marsili-Libelli, S. (1992). Deterministic ad fuzzy control of the sedimentation process, *In: Proceedings Workshop Monitoring, Modelling and Control of the Activated Sludge Process*, Med. Fac. Landbouww Rijksuniv, Gent, 57: 2229-2238.

McKinney, R.E. (1962). Mathematics of Complete Mixing Activated Sludge, J. Sanit. Eng. Div.*, Proceedings of the American Society of Civil Engineers*, 88, SA3.

Nejjari, F., Dahhou, B., Benhammou, A., and Roux, G. (1999). Non-linear multivariable adaptive control of an activated sludge wastewater treatment process. *International Journal of Adaptive Control and Signal Processing*, 13(5): 347-365.

Renard, P., Dochain, D., Bastin, G., Haveau, H., and Nyns, E.-J. (1988). Adaptive control of anaerobic digestion processes – *A pilot-scale application, Biotechnol. Bioeng.,* 31: 287-294.

Roeleveld, P.J., and van Loosdrecht, M.C.M. (2002). Experience with guidelines for waster characterization in The Netherlands. *Water Sci. Technol*., 45(6): 77–87.

Sánchez-Peña, R.S., and Sznaier, M. (1998). Robust Systems Theory and Applications, *John Wiley & Sons*, Inc..

Sollfrank, U., and Gujer, W. (1991). Characterization of domestic wastewater for mathematical modeling of the activated sludge process, *Water Sci. Technol*., 23: 1057-1066.

Yoo, C.K., and Kim, M.H. (2009). Industrial experience of process identification and set-point decision algorithm in a full-scale treatment plant, *Journal of Environmental Management*, 90(8): 2823-2830.