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Adaptive Gain Sliding Observer Based Sliding Controller for Uncertain Parameters Nonlinear Systems. Application to Flexible Joint Robots

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Abstract **– An adaptive gain smooth sliding controller, based on a smooth sliding observer, is developed to control nonlinear SISO affine systems with uncertain parameters and state functions. Furthermore, an adaptively updated parameter term is introduced in the steady state space model of the controlled system in order to obtain useful information despite fault detection. Using a sliding observer with smooth switching function and adaptive gain increases the robustness w.r.t. uncertainties. The adaptive gains smooth sliding observer and controller are designed to fulfill the attractiveness condition on the corresponding switching surfaces. An application to a single arm, flexible joint robot is presented. In order to alleviate chattering in the observer and the controller, a parameterized tangent hyperbolic is used as a switching function, instead of a pure relay one. The gain of the switching function is adaptively updated, depending on the estimation error or on tracking error.**

I. INTRODUCTION

The state and parameter uncertainties in the model of SISO non-linear systems and the deviations of the parameters from their nominal values lead to difficulties in the parameter identification and the state estimation. All of these make absolutely necessary the design of the controller and/or the observer such as the closed loop robustness (stability with small tracking and estimation errors) is achieved. It is well known that the robustness to model parameter uncertainties and external disturbances of the closed loop can be achieved with variable structure controller. Maintaining the system on a sliding surface weakens the influence of the uncertainties in the closed loop performances and quickly leads to an equilibrium point. In [2] an adaptive variable structure controller with parameterized sigmoid as a switching function and adaptive modifications of its amplitude (denoted λ-modification) is used, instead of a pure relay one with constant gain. In this paper, a parameterized tangent hyperbolic function (denoted k-tanh) is used as a switching function to alleviate, or/and eliminate chattering. Decreasing the parameter k in the switching function makes the gain around zero smaller and the unmodelled dynamics are less excited at high frequency. Also, the delay due to the control input calculus and the finite rate of switching can lead to chattering. Using the λ-modification into the gain of the k-tanh switching function, smoothes the response and increases the robustness to structural uncertainties. The adaptive gain is time depending, with the norm of the corresponding sliding surface, as an input. The combinations of variable structure observer-controller for several particular nonlinear systems with application to robot manipulators are presented in [1] and [8]. Results concerning the exponential convergence of adaptive observer under persistent excitation conditions applied to a class of non-linear systems are shown in [5] and [6]. In [7], the persistent excitation condition to the adaptive observer design is relaxed and an extension to non-linear external perturbed systems is presented. A further parameter term, which may be adaptively updated, is considered in order to obtain information about parameter deviations from their nominal values.

The main contributions of this paper concern the smooth sliding observer-controller, the choice of the gains (the gains of linear part and of the variable structure part, respectively), the updating law of variable structure gains, and, finally, the state and tracking error bounds. The main advantage over other existing technique in the flexible joint robot control is that by using adaptive gain, small estimation and tracking errors can be obtained, with an appropriate choice of the initial condition in the updating law. In addition, parameterized smooth switching function keeps the performances in the presence of uncertainties and measurements noise, without infinite rate of switching.

II. SYSTEMS IN ADAPTIVE OBSERVER FORM

Consider the SISO nonlinear system

$$\dot{x} = f(x) + g(x)u + \sum_{i=1}^{p} q_i(x,u)\pi_i, \quad y = h(x) \tag{1}$$

where $x \in \Re^n$, $u, y \in \Re$, $f, g : \Re^n \to \Re^n$, $h : \Re^n \to \Re$, $\pi \in \Re^p$, $\pi = [\pi_1 \ \cdots \ \pi_p]^T$, $q_i : \Re^n x \Re \to \Re^n$.

The following assumptions hold:

A.2.1. $\operatorname{rank}\left[dh(x) \quad dL_f h(x) \cdots \quad d\left(L_f^{n-1} h(x)\right)\right] = n$, $\forall x \in \Re^n$

where $L_f^i h$ is the ith-order Lie derivative of the smooth function h along the vector field f;

A.2.2. Let r be the vector field, which satisfies

$$\left\langle \left[dh \quad \cdots \quad d\left(L_f^{n-1} h\right)\right]^T, r \right\rangle = \left[0 \quad \cdots \quad 1\right]^T \tag{2}$$

and $\left[ad_f^i r, \quad ad_f^j r\right] = 0, \quad 0 \le i, j \le n-1$, where $ad_f^i r$ represents the ith-order Lie bracket $[f, r]$ of the two vector fields f and r;

A.2.3. $[g, ad_f^j r] = 0, \quad 0 \le j-2 \le n-2$;

A.2.4. the vector fields $ad_f^i r, \quad 0 \le i \le n-1$ are complete;

A.2.5. $[q_i, \quad ad_f^i r] = 0, \quad 0 \le i \le p, 0 \le j \le n-2$;

A.2.6.
$$q_i(x,u) = \beta_i(h(x),u)\sum_{j=1}^{n} b_{n-j+1} ad_{(-f)}^{j-1} r(x), \quad 1 \le i \le p \tag{3}$$

Accordingly with Lemma II.1 from [6], there exists a global space diffeomorphism

$$\zeta = T(x) = \begin{bmatrix} h(x) & L_f h(x) & \cdots & L_f^{n-1} h(x) \end{bmatrix}^T, \; T(x_o) = 0 \tag{4}$$

which transforms the system (1) into

$$\dot{\zeta} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix} \zeta + \psi_o(y)u + \sum_{i=1}^{p} \psi_i(y,u)\pi_i$$

$$= A_c\zeta + \psi_o(y)u + \Psi(y,u)\pi \tag{5}$$

$$y = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \end{bmatrix}\zeta = c_c^T \zeta$$

where $\psi_o : \Re \to \Re^n, \Psi_i : \Re \times \Re \to \Re^n, i = 1,\ldots,p$ are smooth functions. Moreover, based on [4] and [6], by using a filtered transformation

$$z = \zeta - M(t)\pi, \tag{6}$$

the system (5) can be transformed in an adaptive observer form

$$\dot{z} = A_c z + \psi_o(y)u + b\beta^T(t)\pi, \quad y = c_c^T z \tag{7}$$

where $M, b,$ and β are expressed hereafter. The matrix $M \in \Re^{nxp}$ can be expressed as $M = \begin{bmatrix} 0 & N^T \end{bmatrix}^T$, with $N \in \Re^{(n-1)xp}$ unique solution of the differential equation

$$\dot{N} = A_N N + B_N \Psi(y,u), \quad N(0) = N \tag{8}$$

where $A_N \in \Re^{(n-1)x(n-1)}, B_N \in \Re^{(n-1)xn}$ are chosen as in [6]. The vector $b \in \Re^n, b = \begin{bmatrix} 1 & b_1, \cdots, & b_n \end{bmatrix}^T$ has constant elements which are the coefficients of Hurwitz polynomial: $s^{n-1} + b_2 s^{n-2} + \cdots + b_n$. Replacing (6) in (8) and using the above notations, the matrix M can be written as the unique solution of the differential equation

$$\dot{M} = (A_c - bc_c^T A_c)M + (I - bc_c^T)\Psi(y,u), \; M(0) = \begin{bmatrix} 0 \\ N(0) \end{bmatrix} \tag{9}$$

The matrix A_N being a Hurwitz matrix, then the matrices $N(t)$ and $M(t)$ are bounded if the control input u and the function $\Psi(y)$ are bounded. The vector $\beta \in \Re^P$ is a continuous bounded function and can be expressed as:

$$\beta(t) = \begin{bmatrix} \beta_1(t), \cdots, & \beta_p(t) \end{bmatrix}^T = c_c^T A_c M + c_c^T \Psi(y,u) \tag{10}$$

Remark 2.1. If the assumptions A.2.2 and A.2.3 hold, then each element $\psi_{oi}(y)u, \quad i = 1,\ldots,n$ is independent of x. If not, some or all $\psi_{oi}(y)$ may depend of z ($\psi_{oi}(z,y)$). In this case the system (7) can be written as

$$\dot{z} = A_c z + \psi_o(z,y)u + b\beta^T(y,u,t)\pi, \quad y = c_c^T z \tag{11}$$

which, obviously, it is not in adaptive observer form.

Remark 2.2. If assumption A.2.6 holds, the system (1) can be transformed directly in the adaptive observer form (7) by using the global diffeomorphism (4), without passing trough the intermediary transformed form (5).

Let ρ be the integer defined as the global relative degree of the system (1). From the definition 4.1.2 of [5], the global relative degree is the integer such that:

$$\begin{aligned} & L_g L_f^i h(x) = 0, \quad \forall x \in \Re^n, 0 \le i \le \rho - 2 \\ & L_g L_f^{\rho-1} h(x) \ne 0, \quad \forall x \in \Re^n \end{aligned} \tag{12}$$

Obviously, the transformed systems (4) and (7) have the same relative degree as the original system (1). Taking into account the relative degree, the elements of the vector term $\psi_o(y)u$, from (7) or (11), can be written as:

$$\begin{aligned} & \psi_{oi}u = 0, \quad i = 1,\ldots,\rho-1, \quad \psi_{oj}u = L_g L_f^{j-1} h(x)u, \\ & j = \rho,\ldots,n-1, \quad \psi_{on}u = L_f^n h(x) + L_g L_f^{n-1} h(x)u \end{aligned} \tag{13}$$

III. ADAPTIVE GAIN SMOOTH SLIDING OBSERVER

The attention is focused on the system (1) and on its transformed form (11). A smooth sliding observer, with constant or adaptive gain is proposed in this section. All the uncertainties are considered on the function f and g. Define the estimates of f and g as $\hat{f}$ $\hat{g}$.

Remark 3.1. The transformed system (11) is more general than the system (7), although the last one is in adaptive observer form.

If $S_o = z_1 - \hat{z}_1$ is chosen as the sliding surface, then we will proof that the first ρ state estimate errors are ultimately bounded, while the others $n - \rho$ errors are bounded in the presence of the model uncertainties into the functions $f(x)$ and $g(x)$. Consider the following assumptions:

A.3.1. The functions f, g, h are all of C^n -class;

A.3.2. The transformation, introduced by (4), is a global diffeomorphism;

A. 3.3. The relative degree of the system, introduced by (12), fulfils the inequality $\rho < n$;

A.3.4. The uncertainties in the functions $f(x)$ and $g(x)$, defined as

$$\begin{aligned} & \Delta_j = \left(L_g L_f^j h(x) - L_{\hat{g}} L_{\hat{f}}^j h(\hat{x})\right)u(\hat{x},t), \; j = \rho-1,\cdots,n-2 \\ & \Delta_n(x,\hat{x},t) = L_f^n h(x) - L_{\hat{f}}^n h(\hat{x}) \\ & + L_g L_f^{n-1} h(x)u(\hat{x},t) - L_{\hat{g}} L_{\hat{f}}^{n-1} h(\hat{x})u(\hat{x},t) \end{aligned} \tag{14}$$

fulfill the following conditions:

$$\left\| \begin{bmatrix} 0 & \cdots & 0 & \Delta_\rho & \cdots & \Delta_{n-1} & 0 \end{bmatrix}^T \right\| < \alpha, \quad \forall t \geq 0 \quad (15)$$

$$\left|\Delta_n(x,\hat{x},t)\right| < \varepsilon\left\|\hat{T}(\hat{x}) - T(x)\right\| + \varphi, \quad \forall t \geq 0 \quad (16)$$

where $\hat{T}(x)$ is a known diffeomorphism which allows the inverse and $\alpha, \varepsilon, \varphi$ are positive constants.

A.3.5. The vector function, $\beta(t) = \beta(y,u,t)$ is uniformly bounded for every bounded pair (y,u).

Theorem 3.1. (Sliding observer convergence). Consider the systems (1), (11) or (7), the last one in the adaptive observer form, $\hat{f}, \hat{g}$ the available estimates of the functions f, g and the uncertainties fulfilling (15), (16), then, one can design the adaptive sliding observer

$$\begin{aligned}
\dot{\hat{z}}_i &= \hat{z}_{i+1} - \gamma_i(\hat{z}_1 - z_1) - \theta_i \tanh[k_o(\hat{z}_1 - z_1)] \\
&+ b_i\beta^T\pi, \quad i = 1,\ldots,\rho-1 \\
\dot{\hat{z}}_j &= \hat{z}_{j+1} - \gamma_j(\hat{z}_1 - z_1) - \theta_j \tanh[k_o(\hat{z}_1 - z_1)] \\
&+ L_{\hat{g}}L_{\hat{f}}^{j-1}h\left(T^{-1}(\hat{z})\right)u + b_j\beta^T\pi, \quad j = \rho,\ldots,n-1 \\
\dot{\hat{z}}_n &= -\gamma_n(\hat{z}_1 - z_1) - \theta_n \tanh[k_o(\hat{z}_1 - z_1)] \\
&+ L_{\hat{f}}^n h\left(T^{-1}(\hat{z})\right) + L_{\hat{g}}L_{\hat{f}}^{n-1}h\left(T^{-1}(\hat{z})\right)u + b_n\beta^T\pi
\end{aligned} \quad (17)$$

having the evolution on a neighborhood of the sliding surface, i.e. $S_o = T_1(x) - \hat{T}_1(\hat{x}) = z_1 - \hat{z}_1 \approx 0$, where k_o is a positive constant. The vector gain $\Gamma = [\gamma_1, \ \cdots, \ \gamma_n]^T$ with the expression

$$\Gamma = -A_c b - \sigma b, \quad \sigma \in \Re_+, \quad (18)$$

can be computed such that $A_c + \Gamma c_c^T$ is a stable matrix. The vector gain $\Theta = [\theta_1, \ \cdots, \ \theta_n]^T$ can be chosen such that the error in the first ρ transformed state estimates is ultimately bounded by an arbitrarily small constant and the error of the other $n-\rho$ states is bounded.

Proof: The dynamics of the state estimation error is

$$\begin{aligned}
\dot{\tilde{z}}_i &= \tilde{z}_{i+1} - \gamma_i\tilde{z}_1 - \theta_i \tanh(k_o\tilde{z}_1), \quad i = 1,\ldots,\rho-1 \\
\dot{\tilde{z}}_j &= \tilde{z}_{j+1} - \gamma_j\tilde{z}_1 - \theta_j \tanh(k_o\tilde{z}_1) - L_g L_f^{j-1}h(x)u \\
&+ L_{\hat{g}}L_{\hat{f}}^{j-1}h(\hat{x})u, \quad j = \rho,\ldots,n-1 \\
\dot{\tilde{z}}_n &= -\gamma_n\tilde{z}_1 - \theta_n \tanh(k_o\tilde{z}_1) - L_f^n h(x) \\
&- L_g L_f^{n-1}h(x)u + L_{\hat{f}}^n h(\hat{x}) + L_{\hat{g}}L_{\hat{f}}^{n-1}h(\hat{x})u
\end{aligned} \quad (19)$$

With the gain Γ computed as in (18), the polynomial identity holds

$$(s+\sigma)\left(s^{n-1} + b_1 s^{n-2} + \cdots + b_n\right) = s^n + \gamma_1 s^{n-1} + \cdots + \gamma_n \quad (20)$$

which, due to the n-1 zero-pole cancellations, leads to the first order strictly real positive transfer function $c_c^T\left[sI - \left(A_c + \gamma c_c^T\right)\right]^{-1} b = (s+\sigma)^{-1}$. Because $A_c + \Gamma c_c^T$ is a stable matrix which fulfils the strictly positive real condition (condition B.1.2 from [5]), the Meyer-Kalman-Yacubovich Lemma B.2.2 and the Theorem B.2.2, [5], may be applied. Consequently, the linear part of the state estimation dynamics, from (18), is globally asymptotically stable, i.e.

$$\lim_{t\to\infty}\left\|z(t) - \hat{z}(t)\right\| = 0, \quad \forall z_o \in \Re^n, \quad \pi \in \Re^p.$$

The gain θ_1 has to be chosen such that the sliding condition is fulfilled (the attractiveness condition) $S_o\dot{S}_o < 0$. This condition leads to

$$\theta_1 + \gamma_1\left|z_1 - \hat{z}_1\right| > \left|z_2 - \hat{z}_2\right|, \quad \forall t \geq 0 \quad (21)$$

Choosing the gain θ_1 such as $\theta_1 > \left|\hat{z}_2 - z_2\right| \quad \forall t \geq 0$, the inequality (21) is satisfied with the equivalent state error dynamics

$$\begin{aligned}
\dot{\tilde{z}}_i &= \tilde{z}_{i+1} - \frac{\theta_i}{\theta_1}\tilde{z}_2, \quad i = 1,\ldots,\rho-1 \\
\dot{\tilde{z}}_j &= \tilde{z}_{j+1} - \frac{\theta_j}{\theta_1}\tilde{z}_2 - L_g L_f^{j-1}h(x)u(\hat{x},t) \\
&+ L_{\hat{g}}L_{\hat{f}}^{j-1}h(\hat{x})u(\hat{x},t), \quad j = \rho,\ldots,n-1 \\
\dot{\tilde{z}}_n &= -\frac{\theta_n}{\theta_1}\tilde{z}_2 - L_f^n h(x) - L_g L_f^{n-1}h(x)u(\hat{x},t) \\
&+ L_{\hat{f}}^n h(\hat{x}) + L_{\hat{g}}L_{\hat{f}}^{n-1}h(\hat{x})u(\hat{x},t)
\end{aligned} \quad (22)$$

The gains θ_i can be chosen such that the following polynomial identity holds

$$s^{n-1} + \frac{\theta_2}{\theta_1}s^{n-2} + \ldots + \frac{\theta n}{\theta_1} = (s+\theta)^{n-1} \quad (23)$$

where θ is a positive constant. Changing the variables

$$\tilde{z}_i = \left[\nu_2 \quad \cdots\theta^{i-2}\nu_i \cdots \quad \theta^{n-2}\nu_n\right]^T, \quad i = 2,\ldots,n \quad (24)$$

and taking into account (16), leads to the last $n-1$ error equations from (17)

$$\dot{\nu} = \theta D\nu + \begin{bmatrix} 0 & \cdots & 0 & \dfrac{\Delta_\rho}{\theta^{\rho-2}} & \cdots & \dfrac{\Delta_n}{\theta^{n-2}} \end{bmatrix}^T \quad (25)$$

$$D = \begin{bmatrix} -C_{n-1}^1 & 1 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ -C_{n-1}^{n-2} & 0 & \ddots & 1 \\ -C_{n-1}^{n-1} & 0 & \ldots & 0 \end{bmatrix} \in \Re^{(n-1)x(n-1)} \quad (26)$$

Define the Lyapunov function $V = \nu^T P\nu$ with the positive definite matrix $P \in \Re^{(n-1)x(n-1)}$ as the solution of the equation $PD + D^T P = -I$. Expressing its derivative

$$\begin{aligned}
\dot{V} &= -\theta\|\nu\|^2 + 2\nu^T P[0 \quad \ldots \quad 0 \quad 1]^T \frac{\Delta_n}{\theta^{n-2}} \\
&+ 2\nu^T P\begin{bmatrix} 0 & \ldots & 0 & \dfrac{\Delta_\rho}{\theta^{\rho-2}} & \ldots & \dfrac{\Delta_{n-1}}{\theta^{n-3}} & 0 \end{bmatrix}^T
\end{aligned} \quad (27)$$

using (15), (16), the definition of ν and the inequality $\nu^T P\nu \leq \lambda_{\max P}\|\nu\|$, the following inequality holds

$$\dot{V} \le -\theta\|\nu\|^2 + 2\|\nu\|\|P_n\|\left(\frac{\varphi}{\theta^{n-2}} + \varepsilon\|\nu\|\right) + 2\|\nu\|\frac{\lambda_{\max P}\alpha}{\theta^{\rho-2}} \quad (28)$$

where P_n is the last column of matrix P. By adding $\mu\|\nu\|^2, \mu > 0$ to both sides of (28), one obtains the second order inequality

$$\begin{aligned}\dot{V} + \mu\|\nu\|^2 &\le -(\theta - 2\varepsilon\|P_n\| - \mu)\|\nu\|^2 \\ &+ 2\frac{1}{\theta^{\rho-2}}\left(\varphi\frac{\|P_n\|}{\theta^{n-2}} + \lambda_{\max P}\alpha\right)\|\nu\|\end{aligned} \quad (29)$$

which is satisfied for

$$\|\nu\| \ge \frac{2}{\theta^{\rho-2}}\frac{\varphi\|P_n\| + \lambda_{\max P}\alpha\theta^{n-\rho}}{(\theta - 2\varepsilon\|P_n\| - \mu)\theta^{n-\rho}}.$$

Due to the Corollary 5.3 of Theorem 5.1 from [3], there exists a finite time t_1 such that

$$\|\nu(t)\| \le \frac{2}{\theta^{\rho-2}}\sqrt{\frac{\lambda_{\max P}}{\lambda_{\min P}}}\frac{\varphi\|P_n\| + \lambda_{\max P}\alpha\theta^{n-\rho}}{(\theta - 2\varepsilon\|P_n\| - \mu)\theta^{n-\rho}}, \forall t \ge t_1 \quad (30)$$

The definition of the vector ν, from (24), and the above inequality lead to the conclusion that

$$\begin{aligned}|\tilde{z}_i(t)| &= \left|\hat{T}_i(\hat{x}) - T_i(x)\right| \le \theta^{i-2}\|\nu\| \le \\ &\le 2\frac{\theta^{i-2}}{\theta^{\rho-2}}\sqrt{\frac{\lambda_{\max P}}{\lambda_{\min P}}}\frac{\varphi\|P_n\| + \lambda_{\max P}\alpha\theta^{n-\rho}}{(\theta - 2\varepsilon\|P_n\| - \mu)\theta^{n-\rho}}, \quad (31) \\ &\forall t \ge t_1, \quad i = 2, \ldots, n\end{aligned}$$

Therefore, all the state observation errors in the transformed space converge to a bounded region, and the first ρ errors could be arbitrarily small for sufficiently large θ.

Remark 3.2. The upper bounds of the state estimation errors, given by (31), allow adaptive gains in sliding observer (17). As in [2], the gains $\theta_i, i = 1, \cdots, n$ become time depending, including λ-modification

$$\dot{\theta}_i(t) = -\lambda_{oi}\theta_i(t) - \vartheta_{oi}|\tilde{z}_1(t)|, \quad (32)$$

where $\lambda_{oi}, \vartheta_{oi}$ are positive constants and $\theta_i(t_o)$ fulfils the polynomial identity (23). The dynamics (32) force $\theta_i(t)$ to negative values. They are almost zero when the observer is in the neighbourhood of the sliding surface.

IV. ADAPTIVE GAIN SLIDING CONTROLLER

The sliding controller, presented hereafter, is an extension of the one from [8] to nonlinear affine systems with the relative degree strictly less than the state dimension. Let choose the controller sliding surface as:

$$\hat{S}_c = \sum_{i=1}^{\rho}\xi_i\left(\hat{z}_i - y_r^{(i-1)}\right) \quad (33)$$

with $\xi_\rho = 1$ and $\xi_i, i = 1, \cdots, \rho - 1$ Hurwitz coefficients. The reference to be tracked y_r is assumed to be a C^n function. The expression of the sliding controller

$$u = \left(L_{\hat{g}}L_{\hat{f}}^{\rho-1}h(\hat{x})\right)^{-1}\begin{bmatrix} -L_{\hat{f}}^{\rho}h(\hat{x}) + y_r^{(\rho)} - \sum_{i=1}^{\rho-1}\xi_i\left(\hat{z}_{i+1} - y_r^{(i)}\right) \\ -\phi\hat{S}_c - \eta\tanh\left(k_c\hat{S}_c\right)\end{bmatrix} \quad (34)$$

is derived from the expression of feedback linearization controller. The gain η has to be computed in order to fulfill the attractiveness of the sliding surface. k_c is a positive constant. The derivative of the controller sliding surface (33) is

$$\dot{\hat{S}}_c = -\tilde{z}_2\sum_{i=1}^{\rho}\xi_i\frac{\theta_i}{\theta_1} - \phi\hat{S}_c - \eta\tanh\left(k_c\hat{S}_c\right) \quad (35)$$

where the gain ϕ is chosen to maintain $\hat{S}_c$ bounded during the observer transient. Applying (31) and assuming that the gains θ_i are chosen as (23), the controller attractiveness condition $\hat{S}_c\dot{\hat{S}}_c < 0$ is satisfied if

$$\eta > \frac{2}{\theta^{\rho-2}}\sqrt{\frac{\lambda_{\max P}}{\lambda_{\min P}}}\frac{\varphi\|P_n\| + \lambda_{\max P}\alpha\theta^{n-\rho}}{(\theta - 2\varepsilon\|P_n\| - \mu)\theta^{n-\rho}}\left|\sum_{i=1}^{\rho}\xi_i C_{n-1}^{i-1}\theta^{i-1}\right|$$

The equivalent dynamics during sliding $\hat{S}_c \approx 0$ can expressed as $\sum_{i=1}^{\rho}\xi_i\left(y^{(i-1)} - y_r^{(i-1)}\right) \approx -\sum_{i=1}^{\rho}\xi_i\tilde{z}_i$ .Assuming the gains ξ_i are the coefficients of a polynomial with all stable and real roots, and using the bounds of the state errors (31), the tracking error fulfils the inequality

$$|y(t) - y_r(t)| \le \frac{2}{\xi_1\theta^{\rho}}\sqrt{\frac{\lambda_{\max P}}{\lambda_{\min P}}}\frac{\varphi\|P_n\| + \lambda_{\max P}\alpha\theta^{n-\rho}}{(\theta - 2\varepsilon\|P_n\| - \mu)\theta^{n-\rho}}\sum_{i=1}^{\rho}\xi_i\theta^i \quad (36)$$

for t sufficiently large

Remark 4.1. If the variable structure term in (33) has the gain adaptively updated by λ-modification

$$\dot{\eta}(t) = -\lambda_c\eta(t) - \vartheta_c|S_c|, \quad (37)$$

with λ_c, ϑ_c positive constants, then the tracking error will reduce asymptotically, By choosing the value of the constant k_o greater than k_c, the smooth switching function of the observer is closer to a pure relay than the smooth switching function of the controller. Therefore, the observer converges faster than the controller with small estimate error.

V. APPLICATION TO A FLEXIBLE JOINT ROBOT MANIPULATOR

The dynamic equations of a single link robot arm with a revolute elastic joint (robot with flexible joints) rotating in a vertical plane are

$$\begin{aligned}J_a\ddot{q}_1 + F_a\dot{q}_1 + k(q_1 - q_2) + Mgl\sin q_1 = 0 \\ J_m\ddot{q}_2 + F_m\dot{q}_2 - k(q_1 - q_2) = u, \quad y = q_2\end{aligned} \quad (38)$$

where q_1 and q_2 are the link displacement angle and the rotor (motor shaft) displacement angle, respectively. The link inertia J_a, the motor rotor inertia J_m, the elastic constant k, the mass link M, the gravity constant g, the centre of mass l and the viscous friction coefficients F_a, F_m are positive constant parameters. The control u is the torque delivered by the motor. Choosing as state variables $x_1 = q_1$, $x_2 = \dot{q}_1$, $x_3 = q_2$, neglecting the viscous friction of the arm, considering the position of the motor shaft as the measured output and introducing the vector term $b\beta^T(y,u,t)\pi = b[y \quad u][1 \quad 1]^T$, then the system state equations can be expressed as:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} x_2 \\ -Mgl/J_a \sin(x_1) - k/J_a(x_1 - x_3) \\ x_4 \\ -F_m/J_m x_4 + k/J_m(x_1 - x_3) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1/J_m \end{bmatrix} u + \begin{bmatrix} u+y \\ 3(u+y) \\ 3(u+y) \\ u+y \end{bmatrix} = f(x) + g(x)u + b\beta^T\pi, \quad y = x_3 \tag{39}$$

The following parameters and uncertainties are considered (note that the matching conditions are not fulfilled): $M = 5$, $g = 10$, $l = 0.5$, $k = 200$, $J_a = 1$, $J_m = 0.05$, $F_m = 0.1$, $K_a = 200$, $K_m = 4500$, $B_m = 2$, $\hat{J}_m = 0.06$, $\hat{M}_a = 30$, $\hat{K}_a = 300$, $\hat{K}_m = 4500$, $\hat{B}_m = 1.5$, where: $M_a = \frac{Mgl}{J_a}$, $K_a = \frac{k}{J_a}$, $K_m = \frac{k}{J_m}$, $B_m = \frac{F_m}{J_m}$.

The Lie derivatives, $L_f^i h(x), i = 0,\cdots,n$, are: $h(x) = x_3$, $L_f h(x) = x_4$, $L_f^2 h(x) = -B_m x_4 + K_m(x_1 - x_3)$,
$L_f^3 h(x) = (B_m^2 - K_m)x_4 - B_m K_m(x_1 - x_3) + K_m x_2$
$L_f^4 h(x) = (2B_m K_m - B_m^3)x_4 - M_a K_m \sin(x_1)$
$+ K_m(B_m^2 - K_m - K_a)(x_1 - x_3) - B_m K_m x_2$

The Lie derivatives, $L_g L_f^i h(x), i = 0,\cdots,n-1$, are:

$L_g h(x) = 0$, $L_g L_f h(x) = \frac{1}{J_m}$, $L_g L_f^2 h(x) = -\frac{B_m}{J_m}$,

$L_g L_f^3 h(x) = \frac{B_m^2 - K_m}{J_m}$.

One remarks that the system is of order 4 (n=4) and of relative degree two ($\rho = 2$).

The state transformation, defined in (4), is

$$z = \begin{bmatrix} x_3 \\ x_4 \\ -B_m x_4 + K_m(x_1 - x_3) \\ (B_m^2 - K_m)x_4 - B_m K_m(x_1 - x_3) + K_m x_2 \end{bmatrix} \tag{40}$$

which has the following inverse transform

$$x = T^{-1}(z) = \begin{bmatrix} \frac{z_3 + B_m z_2 + K_m z_1}{K_m} \\ \frac{z_4 + B_m z_3 + K_m z_2}{K_m} \\ z_1 \\ z_2 \end{bmatrix} \tag{41}$$

The transformed state equations are

$$\begin{aligned} &\dot{z}_1 = z_2 + u + y, \quad \dot{z}_2 = z_3 + \frac{1}{J_m}u + 3(u+y) \\ &\dot{z}_3 = z_4 - \frac{B_m}{J_m}u + 3(u+y) \\ &\dot{z}_4 = -M_a K_2 \sin\left(\frac{z_3 + B_m z_2 + K_m z_1}{K_m}\right) - B_m K_a z_2 \\ &-(K_a + K_m)z_3 - B_1 z_4 + \frac{B_m^2 - K_m}{J_m}u + (u+y) \end{aligned} \tag{42}$$

In order to alleviate the chattering in the state estimates and control input, a parameterized tangent hyperbolic will be used as switching function and gain adaptively updated in the observer, as in (32), and in the controller as in (37). Choosing $b = [1 \quad 3 \quad 3 \quad 1]^T$ and $\sigma = 10$, the expression (18) yields $\Gamma = [-13 \quad -33 \quad -31 \quad -10]^T$ for the observer vector gain. With $\theta = 50$, $\theta_1 = 1$, the other sliding observer gains can be obtained, from the polynomial identity (23): $\theta_2 = 150$, $\theta_3 = 7500$ and $\theta_4 = 125000$. The observer is as follows:

$$\begin{aligned} &\dot{\hat{z}}_1 = \hat{z}_2 - \gamma_1\tilde{z}_1 + \theta_1(t)\tanh(k_o\tilde{z}_1) + u + y \\ &\dot{\hat{z}}_2 = \hat{z}_3 - \gamma_2\tilde{z}_1 + \theta_2(t)\tanh(k_o\tilde{z}_1) + \frac{1}{\hat{J}_m}u + 3(u+y) \\ &\dot{\hat{z}}_3 = \hat{z}_4 - \gamma_3\tilde{z}_1 + \theta_3(t)\tanh(k_o\tilde{z}_1) - \frac{\hat{B}_m}{\hat{J}_m}u + 3(u+y) \\ &\dot{\hat{z}}_4 = -\hat{M}_a\hat{K}_m \sin\left(\frac{\hat{z}_3 + \hat{B}_m\hat{z}_2 + \hat{K}_m\hat{z}_1}{\hat{K}_m}\right) \\ &-\gamma_4\tilde{z}_1 + \theta_4(t)\tanh(k_o\tilde{z}_1) - \hat{B}_m\hat{K}_a\hat{z}_2 \\ &-(\hat{K}_a + \hat{K}_m)\hat{z}_3 - \hat{B}_m\hat{z}_4 + \frac{\hat{B}_m^2 - \hat{K}_m}{\hat{J}_m}u + u + y \end{aligned} \tag{43}$$

Note that, if the adaptive gain with λ-modification is used in the sliding observer term, the above values of θ_i become negative initial values of the adaptation law (32).

Accordingly with (33), the controller sliding surface is defined as $\hat{S}_c = \hat{z}_2 - \dot{y}_r + \xi(\hat{z}_1 - y_r)$ with $\xi = 10$. The corresponding sliding control input can be expressed as

$$u = \hat{J}\left[-\hat{z}_3 + \ddot{y}_r - \xi(\hat{z}_2 - \dot{y}_r) - \phi\hat{S} + \eta(t)\tanh(k_c\hat{S}_c)\right] \tag{44}$$

where $\eta(0) = -50$ in the updated law (37). With this value, the attractiveness condition is satisfied. In order to increase the sliding observer convergence and to force the sliding controller state estimates closer to the true ones, the parameter k_o has to be chosen greater than k_c in the corresponding switching function. Therefore, the gain of the tangent hyperbolic switching function is greater around the origin.

The trajectory to be tracked is

$$y_r(t) = 1 + \cos(2t) \quad (45)$$

In the figure 1 the response without chattering can be observed. This is due to the appropriate values of the parameters in the switching functions (the convergence speed of the observer is greater than that of the controller). Small parameter uncertainties (5%) have been considered and random measurement noise. The curves, shown in the figure 2, exhibit a chattering during the observer transient. In this case, the observer convergence rate is comparable with the controller one.

Limitation of the controller amplitude has been introduced. The above values of θ_i, obtained from the polynomial identity (23) have been used as negative initial values in the update law (32).

VI. CONCLUSIONS

A smooth sliding observer-controller with adaptively updated gains for the switching functions is proposed in order to control nonlinear systems. A parameterised tangent hyperbolic function is used as a switching function. The state dynamics of the controlled system include an extra parameter term, further adaptively updated, in order to obtain useful information despite fault detection. The parameterised tangent hyperbolic function assures the alleviation or complete elimination of chattering. An appropriate choice of parameters in the observer and controller switching functions allows the observer convergence before that of the controller. Adaptive gains, with appropriate initial values, lead to small estimation and output tracking error and improve robustness. Convergence rates, both for the observer and the controller have been established. An application to a flexible joint with one rigid link robot control is presented. Closed loop response, obtained by simulation, confirms the theoretical results.

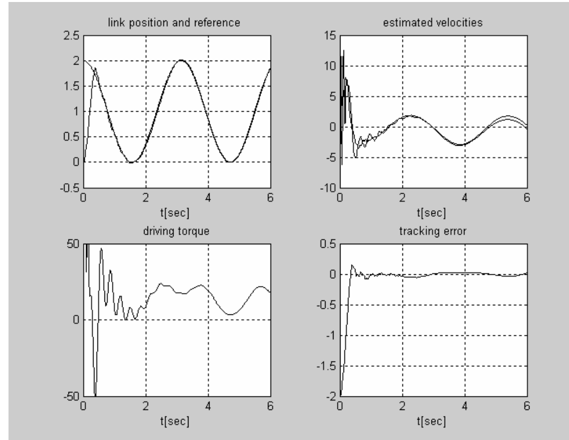

Fig. 1. Closed loop robot response, smooth sliding observer and controller, parameterized tangent hyperbolic switching function $k_o = 0.5$, $k_c = 0.25$, adaptive gains with λ-modification: $\lambda_o = 1, \vartheta_o = 1$ $\lambda_c = 1$, $\vartheta_c = 1$

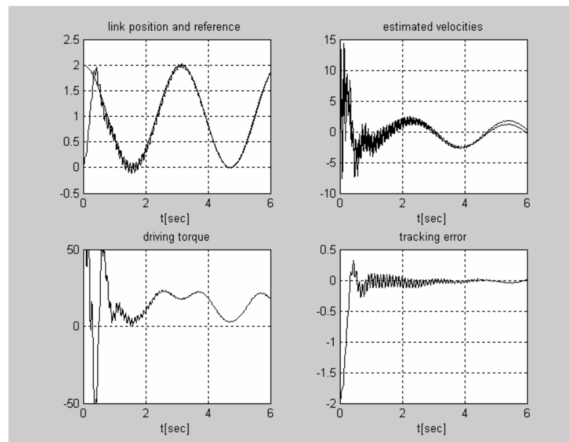

Fig. 2. Closed loop robot response, smooth sliding observer and controller, parameterized tangent hyperbolic switching function $k_o = 50$, $k_c = 50$, adaptive gains with λ-modification: $\lambda_o = 1, \vartheta_o = 1$ $\lambda_c = 1$, $\vartheta_c = 1$.

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